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ABSTRACT

Discriminant analysis is a multivariate method of analysis with two purposes: (1) to describe differences among groups; or (2) to classify participants into groups. Either linear or quadratic rules can be used in both descriptive discriminant analysis (DDA) and predictive discriminant analysis (PDA). In both DDA and PDA the researcher wants to use the rules that are the most accurate and the most replicable, but these two considerations often offer competing perspectives about which rule is optimal in a given analysis. The linear rule is used when group covariance matrices are pooled. The linear rule has greater external generalizability because fewer parameters are estimated. The quadratic rule is used when separate group covariance matrices are used. These covariance matrices do not need to be as homogenous. However, the quadratic rule's external generalizability is not as accurate. It is important to use the correct rule in both PDA and DDA. (Contains 10 references.) (Author/SLD)

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Linear and Quadratic Rules:

What are They and When are They Used?

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Abstract

Multivariate methods have become more commonly used in recent decades (Grimm & Yarnold, 1995). Discriminant analysis is a multivariate methods with two purposed: (1) to describe differences among groups, or (2) to classify participants into groups (Stevens, 1996). Either "linear" or "quadratic" rules can be used in both descriptive discriminant analysis (DDA) and predictive discriminant analysis (PDA). In both DDA and PDA the researcher wants to use rules that are (a) most accurate and (b) most replicable; however, these two considerations often offer competing perspectives regarding which rule is optimal in a given analysis.

The Uses of Linear and Quadratic Rules In Predictive and Descriptive Discriminant Analysis

Multivariate methods have become more common over the past 20 years (Grimm & Yarnold, 1995). Multivariate methods are designed for multiple outcome variables. As Huberty and Morris (1989) noted, multivariate methods ask, "Are there any overall effects present?" This questioning, or this philosophy, best honors the reality from which data are collected. That is, if data are collected from samples upon which there are many influences, or variables, then it is logical to use a statistical method that is designed to take those variables into account (Altman, 2000; Thompson, 1994).

Because multivariate methods are designed for multiple outcome variables, multivariate methods require only one omnibus test to determine if any differences exist. Therefore, multivariate methods not only honor the reality from which the data was collected, but help control the inflation of experimentwise error (Altman, 2000).

Discriminant analysis is a multivariate method that can be used for two distinct purposes: (1) to describe major differences among groups, and (2) to classify participants into groups (Stevens, 1996). Descriptive discriminant analysis (DDA) is used to describe differences among groups; predictive discriminant analysis (PDA) is used to classify participants into groups.

DDA is typically used post hoc to explore the effects first detected by a multivariate analysis of variance (MANOVA). In other words, DDA is commonly used as a post hoc procedure for a

MANOVA (Huberty, 1994). Interpretation of DDA focuses on DDA function coefficients and structure coefficients. Hit rates are irrelevant.

PDA is a technique in which interval response variables are used to predict a given outcome variable (e.g., those who drop out of school verses those who do not drop out of school). Interpretation of PDA focuses on the resulting "hit rate" (Grimm & Yarnold, 1997; Huberty, 1994). Function and structure coefficients are irrelevant.

Classification

Discriminant analysis may be used as a classification procedure. PDA classifies participants according to a set of criteria that distinguishes one group from another. A participant resembles group *k* if the vector of scores for that participant is closest to the vector of means (centroid) for group *k*. Therefore, a participant is closest in a distance sense (Mahalanobis distance) to the centroid of that group (Stevens, 1996). This can be done because the classification rules most often used in discriminant analysis are based on multivariate normal distribution theory (Huberty & Curry, 1978).

Misclassification of data can have harmful results (Klecka, 1980). For example, making the correct classification regarding whether a tumor is malignant or benign can determine whether an individual will receive the correct treatment. Huberty and Curry (1978) stated that a misclassification "occurs when, if an individual is misclassified, he is classified into a population other than on 'closest' to his actual population" (p. 240).

Therefore, when determining which classification rule one will apply, it is important to choose a rule that most accurately classifies individuals into the group they most resemble.

Two rules that are often used in classification procedures are the "linear" and the "quadratic" rules. These two rules are based on the similarity of covariance structure of the predictors across a given criterion population (Huberty & Curry, 1978). Therefore, the decision as to which classification rule to use depends on the similarity of the group covariance matrices (Young, 1993).

The Linear Rule

The linear rule is used if the group covariance matrices are pooled (McGee, 2000). Of course, this pooling of the separate covariance is legitimate only if the covariances and variabilities of the scores are roughly equivalent across the croups (Haase & Thompson, 1992).

Because the linear rule pools the group covariance matrices and does not capitalize greatly on sampling error by estimating as many parameters, it has greater external generalizability (Huberty & Curry, 1978). Of course, this is only if the homogeneity assumption is met. Therefore, the linear rule is best used when the covariance matrices are reasonably similar and when external generalizability is more important.

The Quadratic Rule

The quadratic rule does not pool group covariance matrices. The quadratic rule is based on separate group covariance matrices

for each group. Because there is no pooling of group covariance matrices, the covariance matrices do not have to be homogeneous.

. However, a drawback to this procedure is that the quadratic rule capitalizes on sampling error. When we apply the quadratic rule, we compute a separate variable-by-variable covariance matrix for each of the groups (e.g., 3 groups). Thus, we are estimating a lot of variances and covariances when we use a quadratic rule, as opposed to a linear rule that estimates the entries in a single "pooled" covariance matrix. Because a quadratic rule takes more sampling information into account, the result may be a higher PDA hit rate or DDA effect size in the sample. However, for the same reason (capitalization on sampling error in the sample), this greater PDA hit rate or DDA effect size in ur sample may therefore also be less generalizable to new samples. In other words, when we apply our PDA or DDA rule in a new sample, the PDA hit rate or DDA effect size often will deteriorate substantially more than would results for a linear rule.

When this rule is used it usually increases hit rates when used in PDA and decreases Wilk's lambda in DDA (McGee, 2000). However, because external generalizability is often more important when using PDA, the quadratic rule is not the most effective rule for this procedure.

Summary

When classifying gropus or individuals, it is important to rely on a procdure that is (a) accurate and (b) replicable. If the procedure is not accurate the results can be harmful (Klecka,

1980). The linear rule is used when group covariance matrices are pooled (McGee, 2000). Because the linear rule relies on pooled group covariance matrices, these matrices must be relatively homogeneous. The linear rule has greater external generalizability (Huberty & Curry, 1978) because fewer parameters are estimated. The quadratic rule is used when separate group covariance matrices are employed. Therefore, these covariance matrices do not need to be as homogeneous. However, the quadratic rule does not have as accurate external generalizability. It is critical to use the right rule in both PDA and DDA.

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